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SCALE-UP IN POROELASTIC SYSTEMS AND APPLICATIONS TO RESERVOIRS

James G. Berryman [1]

ABSTRACT

A fundamental problem of heterogeneous systems is that the macroscale behavior is not necessarily well-described by equations familiar to us at the meso- or microscale. In relatively simple cases like electrical conduction and elasticity, it is true that the equations describing macroscale behavior take the same form as those at the microscale. But in more complex systems, these simple results do not hold. Consider fluid flow in porous media where the microscale behavior is well-described by Navier-Stokes' equations for liquid in the pores while the macroscale behavior instead obeys Darcy's equation. Rigorous methods for establishing the form of such equations for macroscale behavior include multiscale homogenization methods and also the volume averaging method. In addition, it has been shown that Biot's equations of poroelasticity follow in a scale-up of the microscale equations of elasticity coupled to Navier-Stokes. Laboratory measurements have shown that Biot's equations indeed hold for simple systems but heterogeneous systems can have quite different behavior. So the question arises whether there is yet another level of scale-up needed to arrive at equations valid for the reservoir scale? And if so, do these equations take the form of Biot's equations or some other form? We will discuss these issues and show that the double-porosity equations play a special role in the scale-up to equations describing reservoir behavior, for fluid pumping, geomechanics, as well as seismic wave propagation.

Keywords: poroelasticity, double porosity, effective medium theory, up-scaling

INTRODUCTION

Earth materials composing either aquifers or oil and gas reservoirs are generally heterogeneous, porous, and often fractured or cracked. Distinguishing water, oil, and gas using seismic signatures is a key issue in seismic exploration and reservoir monitoring. Traditional approaches to seismic monitoring have often used Biot's theory of poroelasticity (Biot 1941; Biot 1956a; Biot 1956b; Biot 1962; Gassmann 1951). Many of the predictions of this theory, including the existence of the slow compressional wave, have been confirmed by both laboratory and field experiments (Plona 1980; Berryman 1980; Johnson et al. 1982; Chin et al. 1985; Winkler 1985; Pride and Morgan 1991; Thompson and Gist 1993; Pride 1994).

Rather than trying to deal with all the heterogeneity at once, we choose to consider a model intended to capture two main features of importance. Just two types of porosity are often key at the reservoir scale: (1) Matrix porosity occupies a finite and substantial fraction of the volume of the reservoir. This porosity is often called the storage porosity since it stores the fluids of interest. (2) Fracture or crack porosity may occupy very little volume overall, but nevertheless has two very big effects on reservoir behavior. First the fractures/cracks drastically weaken the

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rock mechanically, so that a change in a very low effective stress level may introduce nonlinear geomechanical responses. The second effect is that fractures/cracks introduce a fast pathway for the fluid to escape from the reservoir. This effect is obviously key to reservoir analysis and the economics of fluid withdrawal.

Many attempts have been made to incorporate fractures into rock models, and especially models that try to account for compressional wave attenuation in rocks containing fluids. But these models have often been viscoelastic rather poroelastic (Budiansky and O'Connell 1976; O'Connell and Budiansky 1977). Berryman and Wang (1995) showed how to make a rigorous extension of Biot's poroelasticity to include fractures/cracks by making a generalization to double-porosity/dual-permeability media modeling. That work concentrated on geomechanics and fluid flow aspects of the problem in order to deal with the interactions between fluid withdrawal and elastic closure of fractures during reservoir drawdown. The resulting equations were later applied to the reservoir consolidation problem by Lewallen and Wang (1998). Berryman and Wang (2000) then showed how the double-porosity approach could be applied to wave propagation problems, thereby generalizing Biot's work on waves to allow for heterogeneous porosities and permeabilities.

The present paper addresses the question of scale-up in heterogeneous reservoirs. If Biot's equations of poroelasticity are the correct equations at the mesoscale, then what are the correct equations at the macroscale? We show that Biot's equations are not the correct equations at the macroscale when there is significant heterogeneity in fluid permeability. However, the double-porosity dual-permeability approach appears to permit consistent modeling of such reservoirs and also shows that no further up-scaling is required beyond the double-porosity stage.

EQUATIONS OF BIOT'S SINGLE-POROSITY POROELASTICITY

For long-wavelength disturbances ($\lambda >> h$, where h is a typical pore size) propagating through a single-porosity porous medium, we define average values of the (local) displacements in the solid and also in the saturating fluid. The average displacement vector for the solid frame is $\mathbf{u}$, while that for the pore fluid is $\mathbf{u}_f$. The average displacement of the fluid relative to the frame is $\mathbf{w} = \phi(\mathbf{u} - \mathbf{u}_f)$. For small strains, the frame dilatation is e, while the increment of fluid content is defined by

$$\zeta = -\nabla \cdot \mathbf{w} = \phi(e - e_f). \tag{1}$$

With time dependence of the form $\exp(-i\omega t)$, the coupled wave equations that follow in the presence of dissipation are

$$\begin{aligned} -\omega^2(\rho\mathbf{u} + \rho_f\mathbf{w}) &= H\nabla e - C\nabla\zeta + \mu_d\left(\nabla^2\mathbf{u} - \nabla e\right), \\ -\omega^2(\rho_f\mathbf{u} + q\mathbf{w}) &= C\nabla e - M\nabla\zeta = -\nabla p_f, \end{aligned} \tag{2}$$

where μ_d is the drained shear modulus, H, C, and M are bulk moduli,

$$\rho = \phi\rho_f + (1-\phi)\rho_m, \tag{3}$$

and

$$q = \rho_f\left[\alpha/\phi + iF(\xi)\eta/\kappa\omega\right]. \tag{4}$$

The kinematic viscosity of the liquid is η; the permeability of the porous frame is κ; the dynamic viscosity factor is given approximately [or see Johnson et al. (1987) for more discussion], for our choice of sign for the frequency dependence, by

$$F(\xi) = \tfrac{1}{4}\{\xi T(\xi)/[1 + 2T(\xi)/i\xi]\}, \tag{5}$$

where

$$T(\xi) = \frac{\mathrm{ber}'(\xi) - i\mathrm{bei}'(\xi)}{\mathrm{ber}'(\xi) - i\mathrm{bei}'(\xi)} \tag{6}$$

and

$$\xi \equiv (\omega/\omega_0)^{\frac{1}{2}} = (\omega\alpha\kappa/\eta\phi)^{\frac{1}{2}} = (\omega h^2/\eta)^{\frac{1}{2}}. \tag{7}$$

The functions $\mathrm{ber}(\xi)$ and $\mathrm{bei}(\xi)$ are the real and imaginary parts of the Kelvin function. The dynamic parameter h is a characteristic length generally associated with and comparable in magnitude to the steady-flow hydraulic radius. The tortuosity $\alpha \geq 1$ is a pure number related to the frame inertia which has been measured (Johnson et al. 1982) and has also been estimated theoretically (Berryman 1980; Berryman 1983).

The coefficients H, C, and M are given by (Gassmann 1951; Geertsma 1957; Biot and Willis 1957; Geertsma and Smit 1961; Stoll 1974)

$$H = K_d + \frac{4}{3}\mu_d + (1 - K_d/K_m)^2 M, \tag{8}$$

$$C = (1 - K_d/K_m)M, \tag{9}$$

where

$$M = 1/[(1 - \phi - K_d/K_m)/K_m + \phi/K_f]. \tag{10}$$

The constants are drained bulk and shear moduli K_d and μ_d, mineral bulk modulus K_m, and fluid bulk modulus K_f. Korringa (1981) showed equations (8)-(10) to be correct as long as the porous material may be considered homogeneous on the microscopic scale as well as the macroscopic scale. Also, see a recent tutorial on Gassmann's equations (Gassmann 1951) by Berryman (1999).

To decouple the wave equations (2) into Helmholtz equations for the three modes of propagation, we note that the displacements $\mathbf{u}$ and $\mathbf{w}$ can be decomposed as

$$\mathbf{u} = \nabla\Upsilon + \nabla \times \vec{\beta}, \qquad \mathbf{w} = \nabla\psi + \nabla \times \vec{\chi}, \tag{11}$$

where Υ, ψ are scalar potentials and $\vec{\beta}$, $\vec{\chi}$ are vector potentials. Substituting (11) into (2), we find (2) is satisfied if two pairs of equations are satisfied:

$$(\nabla^2 + k_s^2)\vec{\beta} = 0, \qquad \vec{\chi} = -\rho_f\vec{\beta}/q \tag{12}$$

and

$$(\nabla^2 + k_\pm^2)A_\pm = 0. \tag{13}$$

The wavenumbers in (12) and (13) are defined by

$$k_s^2 = \omega^2(\rho - \rho_f^2/q)/\delta\mu \tag{14}$$

and

$$k_\pm^2 = \tfrac{1}{2}\left[b + f \mp \left[(b-f)^2 + 4cd\right]^{\frac{1}{2}}\right], \tag{15}$$

$$b = \omega^2(\rho M - \rho_f C)/\Delta, \qquad c = \omega^2(\rho_f M - qC)/\Delta,$$
$$d = \omega^2(\rho_f H - \rho C)/\Delta, \qquad f = \omega^2(qH - \rho_f C)/\Delta, \tag{16}$$

with

$$\Delta = HM - C^2. \tag{17}$$

The linear combination of scalar potentials has been chosen to be

$$A_\pm = \Gamma_\pm \Upsilon + \psi, \tag{18}$$

where

$$\Gamma_\pm = d/(k_\pm^2 - b) = (k_\pm^2 - f)/c. \tag{19}$$

With the identification (19), the decoupling is complete.

LOW FREQUENCY ASYMPTOTICS FOR SINGLE-POROSITY

We will first demonstrate the dichotomy of interest by showing what Biot's theory predicts if it is applied to heterogeneous reservoirs. The main issues with up-scaling in poroelasticity occur for the low frequency asymptotics, and so we limit discussion to this regime here. For low frequencies, all the wavelengths are long, thereby covering large regions of the heterogeneous medium, and so up-scaling is an issue that must always be addressed in this limit.

Compressional and Shear Waves

Compressional and shear waves have almost the same asymptotic behavior at low frequencies, but the analysis for shear waves is much shorter, so we will present only the shear wave analysis here.

The wavenumber k_s for shear wave propagation is determined by (14), and when $\omega \to 0$ we have $q \to i\rho_f\eta/\kappa\omega$, so

$$k_s^2 = \frac{\omega^2\rho}{\mu_d}\left[1 + i\frac{\rho_f\kappa\omega}{\rho\eta}\right]. \tag{20}$$

Thus, when the loss tangent is a small number, we find the shear wave quality factor is

$$1/Q_s \simeq \frac{\rho_f\kappa\omega}{\eta\rho}. \tag{21}$$

Total attenuation along the path of a shear wave is then determined by the integral $\int \frac{\rho_f\kappa\omega^2}{2\eta(\rho\mu)^{1/2}} d\ell$ along the path of the wave. We assume for the sake of argument that the fluid is the same throughout the reservoir. So all fluid factors as well as frequency are constant. The solid material parameters μ_d and ρ_m and also the porosity ϕ (which is hidden in ρ) may vary in the reservoir, but these variations will be treated here as negligible compared the variations in the permeability κ. Thus, we find that the total attenuation along a path of length $L = \int d\ell$ is approximately proportional to $\int \kappa d\ell$. The average attenuation per unit length of the travel path is therefore proportional to $\int \kappa d\ell/L$, which is just the mean of the permeability along the wave's path. This result is also true for the compressional waves, but the other multiplicative factors are a bit more complicated in that case.

Slow Waves

In contrast, the slow compressional wave can have two very different types of behavior at low frequency depending on the magnitude of the permeability. The wavenumber k_- for slow wave propagation is determined by (15). To simplify this equation, we note that it is an excellent approximation to take

$$k_-^2 \simeq b + f = \frac{\omega^2}{\Delta}\left[qH - 2\rho_f C + \rho M\right]. \tag{22}$$

So, at low frequencies, k_-^2 is proportional to q, whereas k_s^2 was inversely proportional to q. Then, for small frequencies but large values of the permeability, $q \rightarrow \rho_f[\alpha/\phi + i\eta/\kappa\omega]$. Substituting this into (22), we find that

$$k_-^2 = \frac{\omega^2}{\Delta}\left[\alpha\rho_f H/\phi - 2\rho_f C + \rho M + i\eta\rho_f H/\kappa\omega\right]. \tag{23}$$

So as $\omega \rightarrow 0$ for large κ, there will be an intermediate frequency regime in which the slow wave has a well-defined quality factor

$$1/Q_- \simeq \eta\rho_f H/\kappa\omega(\alpha\rho_f H/\phi - 2\rho_f C + \rho M), \tag{24}$$

which for strong frame materials reduces to

$$1/Q_- \simeq \eta\phi/\alpha\kappa\omega. \tag{25}$$

Except for some factors of density, porosity, and tortuosity, this expression is essentially the inverse of the corresponding expression for $1/Q_s$. Obviously both factors cannot be small simultaneously except for a very limited range of frequencies, which is determined by the factor $\alpha\rho/\phi\rho_f$. Although the tortuosity $\alpha \geq 1$ in general it can have a wide range of values, for granular media it is typical to find $\alpha \simeq 2$ or 3. In addition, α is also scale invariant, *i.e.*, it does not depend on the size of the particles composing the granular medium. So, the presence of α multiplying κ in (25) does not change the fact that the slow-wave attenuation is strongly influenced by fluctuations in the permeability κ. Being proportional to the square of the typical particle sizes, the permeability is itself not scale invariant. There is nevertheless a fairly small range of frequencies in which the approximation in (25) is valid, say from about 20 kHz to a few MHz for κ's on the order of 1 D ($\simeq 10^{-12}$ m^2). This is the range where a propagating slow wave might be expected to be seen, and in fact has been observed in laboratory experiments (Plona 1980).

For still smaller permeabilities or smaller frequencies or both, the leading approximation for the slow wave dispersion is instead given by

$$k_-^2 \simeq i\frac{\omega\eta\rho_f H}{\kappa\Delta}. \tag{26}$$

This type of dispersion relation corresponds to a purely diffusive process having a diffusion coefficient $\mathcal{D} \simeq M\kappa/\eta\rho_f$. This result follows directly from the second equation in (2) when the porous frame is sufficiently rigid.

We reach the same conclusion about how fluctuating permeability affects the propagation or diffusion of increments of fluid content (*i.e.*, masses of excess fluid particles) in both of these cases. For the wave propagation situation of (25), we clearly have, by simple analogy to the arguments given already, that the average attenuation per unit length along the wave's path is proportional to $\int \kappa^{-1} d\ell/L$. Similarly, in the limit of the diffusion process described by (26), then for a planar excitation diffusing through such a system in a direction perpendicular to the bedding planes, or for regions of isotropic random fluctuations in permeability, we again expect the overall effective diffusion rate to depend on the same average quantity: $\int \kappa^{-1} d\ell/L$. Thus, measurements of slow waves or of fluid increment diffusion on the macroscale will measure an effective permeability that is largely controlled by the smallest permeability present in the system. Clearly, this is exactly the opposite dependence we found for the dependence of the shear wave and also for the fast compressional wave, and must cause difficulties for up-scaling in Biot's theory, where only one permeability parameter is available for the fitting of data.

Discussion

These observations show that there is a significant problem with up-scaling Biot's theory, *i.e.*, that the resulting system of equations is no longer of the same form as Biot's theory. This is certainly no failing of Biot's theory, but rather a failing of any attempted application of Biot's theory directly to the up-scaled macro-system. Biot's theory predicts correctly that compressional and shear wave attenuation both depend on the integral of the permeability κ along the path of each wave. But the permeability itself along the same path averages as the inverse of the permeability (harmonic mean). Thus, the overall permeability depends most strongly on the smallest permeabilities present in the system, while the wave attenuation depends most strongly on the largest permeabilities in the system (Berryman 1988). When we try to up-scale under these circumstances, we have an inherent problem due to the fact that Biot's theory contains only one permeability; yet, for heterogeneous systems, there are two very distinct measures of permeability (the mean and the harmonic mean) that play significant roles.

SUMMARY OF DOUBLE-POROSITY WAVE PROPAGATION ANALYSIS

Berryman and Wang (2000) provide a formulation as well as some specific examples of the predictions of a double-porosity dual-permeability model for wave propagation in heterogeneous poroelastic media. The analysis is fairly tedious and we do not have space to present details here. The main conclusion of the double-porosity analysis is that the presence of the two porosities and permeabilities leads to new modes of propagation. In particular, bulk compressional and shear waves very similar to those in Biot's single-porosity formulation are found, and now there are also two slow compressional waves. As the choices of parameters are varied, there are many types of interactions among these waves that are possible, but in the simplest cases the slow waves individually act like the ones described in the preceding section.

Two Slow Waves

We assume that the two permeabilities in the double-porosity model differ greatly in magnitude so that $\kappa_1 >> \kappa_2$ and that the corresponding porosities satisfy $0 < \phi_1 << \phi_2$. Thus, the first porosity type is transport-like and the second is storage-like. The analysis of the preceding section of the present paper would suggest that the smaller of the two permeabilities would result in a diffusive mode at all frequencies and the larger of the two would result in a propagating slow wave at high frequencies while then degenerating into another diffusive mode at low frequencies. This behavior is exactly what was found in the numerical examples presented by Berryman and Wang (2000).

Shear and Compressional Waves

Shear waves were not studied explicitly by Berryman and Wang (2000), but equation (5) of that paper can be used for that purpose simply by applying the *curl* operator to all three of the equations in the set. When this is done, the result is that the first equation describes the actual shear mode, while the other two equations provide constraints on the relative motion of the pore fluid in each type of porosity versus the displacement of the solid frame. In particular, the shear components of the differences in fluid and solid displacements can be uniquely related by complex factors (that are known explicitly) to the displacement of the solid alone. Furthermore, as in the case for single-porosity poroelasticity, all of the interesting behavior of the shear mode comes from the inertial terms. The form of the resulting dispersion relation at low frequencies is identical to (20) with the replacement

$$\kappa \rightarrow \kappa_1 + \kappa_2 \simeq \kappa_1, \tag{27}$$

since we assume here that $\kappa_1 >> \kappa_2$. A similar result follows for the compressional wave. Thus, as for single-porosity, the attenuation of the shear and compressional waves is dominated by the largest permeability present in the system. However, this leads to no contradiction in the double-porosity formulation. Thus, the problem inherent in up-scaling with single-porosity poroelasticity is resolved in an intellectually satisfying way in the double-porosity approach.

CONCLUSIONS

It is well-known that fluid flow in porous media is well-described at the microscale by Navier-Stokes' equations for fluids in the pores but at the macroscale the behavior instead obeys Darcy's equation. Rigorous methods for establishing the form of such equations for macroscale behavior include multiscale homogenization methods and also the volume averaging method. In particular, it has been shown that Biot's equations of single-porosity poroelasticity follow in a scale-up of the microscale equations of elasticity coupled to Navier-Stokes (Burridge and Keller 1981).

We have found that the equations of single-porosity poroelasticity are not the correct equations at the macroscale when there is significant heterogeneity in fluid permeability. However, the double-porosity dual-permeability approach appears to permit consistent modeling of such reservoirs and also shows that no further up-scaling is required beyond the double-porosity stage in many circumstances.

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